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FAST TRACK COMMUNICATION

Force focusing in confined fibres and sheets

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Online at stacks.iop.org/JPhysD/41/132003**Abstract**

A thin fibre or sheet curled into a circular container has a detached region whose shape and force ratios are independent of material properties and container radius. We compute this shape and compare it with experiments. The discrete forces acting at either end of the detached region have a ratio that depends only on the length of the fibre or sheet relative to the circle radius. We calculate this force ratio in three regimes of circle radius.

A sheet of office paper coiled into a mailing tube hugs the wall of the tube in order to minimize its bending. But the contact with the wall is incomplete; near the edge, the paper detaches or takes off from the wall and rejoins the cylinder only at the edge. Such detachment is a commonplace feature of coiled sheets or fibres small and large. Here we show that the detached region has a universal shape that touches down at an angle of 24.1° . Moreover, the takeoff point experiences a focused force controlled by the length of the fibre or the sheet.

To explain [1] this behaviour, we consider a fibre confined in a circular ring of radius R ; the same reasoning applies to a sheet confined in a cylinder. The detached region is necessary because the fibre must be uncurved at the touchdown point. To bend the fibre at any point requires an external torque around that point [2]. However, the external force P at the touchdown point exerts no torque. Since the end of the fibre is uncurved, it must angle inwards in order to remain in the ring. Thus a segment of some length L must be detached. The universal properties reported below arise from the mechanical equilibrium of this detached segment.

The fibre curves so as to minimize its mechanical energy. We describe the fibre shape by giving its deflection angle $\phi(s)$ from the horizontal at a distance s from the takeoff point. This point $s = 0$ is at angle β to the vertical axis. (figure 1(a)). The end of the fibre, at the bottom of the circle, is at distance $s = L$ from the takeoff point, and $\phi(L) = \alpha$. The upward touchdown

force P acts normal to the circle; thus, frictional forces are not included. The energy per unit length then consists of the deformation energy, proportional to the square of the curvature $d\phi/ds$, and the work done against P . The energy E is thus given by $E = \int_0^L ds [\frac{1}{2}B(d\phi/ds)^2 + P \sin(\phi)]$, where B is the bending stiffness of the fibre [2]. The $\phi(s)$ that minimizes such an energy must satisfy the Euler–Lagrange equation [3]: $Bd^2\phi/ds^2 - P \cos \phi = 0$. The fibre at the takeoff point has the same angle as the adjacent fibre lying against the circle, so that $\phi(0) = \beta$. Further, the curvature must be continuous at $s = 0$: $d\phi/ds|_0 = -1/R$. (Any discontinuity would require an external torque at the takeoff point.) The value of P must be chosen to make the end of the fibre lie on the constraining circle. These constraints uniquely determine the fibre shape [2].

The detached region subtends an angle β , which is independent of the nature of the fibre or the size of the ring. That is, it is independent of the bending stiffness B and the confining radius R . Indeed, there is no way to combine B , which involves energy, with R , which does not, in a way that gives a dimensionless number. Likewise, the touchdown angle α , the relative arc length L/R and the shape of the detached region are independent of B and R (figure 1(b)).

The ring exerts a discrete kickoff force denoted T between fibre and ring at the takeoff point. The tensile counterpart of this force is familiar in de-lamination of multilayer sheets [4]. This kickoff force is needed to balance the touchdown force P . The detached region experiences a tangential pushing force from the remainder of the fibre. However P also has a

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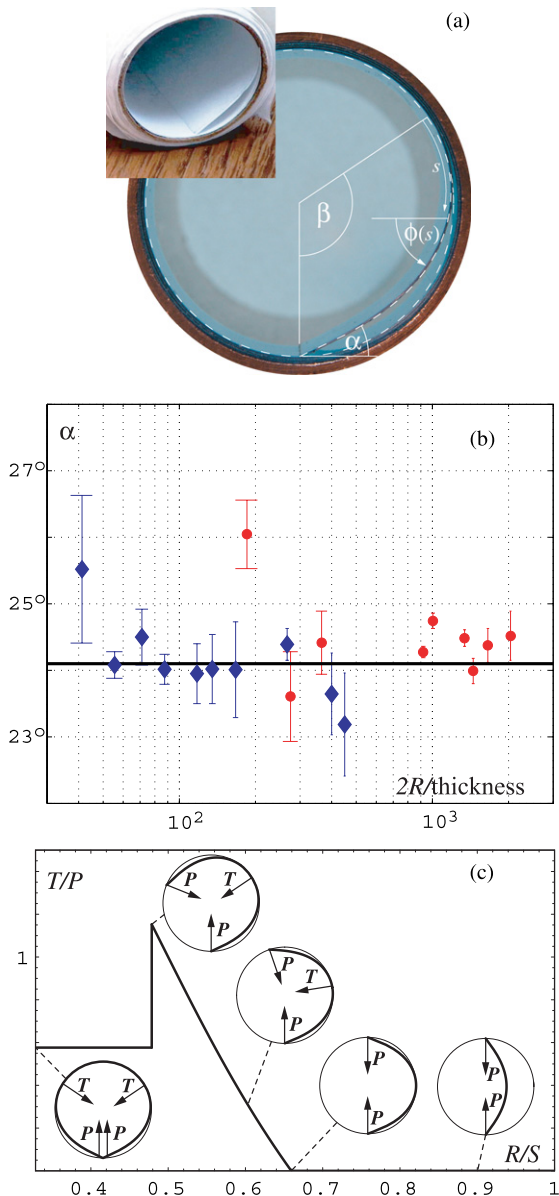


Figure 1. Inset: sheet of office paper coiled in a cardboard tube showing the detachment from the tube at the upper right and touchdown at the bottom. (a) End view photograph of a long, 0.02 cm thick mica strip (black line overlaid with dashed white line) coiled inside a 4.4 cm diameter cylinder. The white dashed line shows predicted shape of the detached region. Subtended angle β , position variable s , angle $\phi(s)$ and touchdown angle α are indicated. (b) Measured touchdown angle α for different materials and confining radii R , illustrating universality of α . \bullet : amorphous metal ribbon of thickness 0.002 cm and width 0.5 cm; \blacklozenge : mica strip of thickness 0.02 cm and width 1 cm. Upper error bound indicates the angle at the contact point; lower error bound is the angle extrapolated from the inner surface of the strip to the boundary. The thick horizontal line indicates the predicted universal value of α . (c) Takeoff force T relative to touchdown force P versus confining radius R , scaled by half-length S . T drops by a factor 2 at the point where two takeoff points appear.

(This figure is in colour only in the electronic version)

component normal to the takeoff surface, and the ring must supply this normal force. Thus $T = -P \cos \beta$, so that $T/P = 0.5763 \dots$; the fibre functions as a precision force divider.

Short fibres behave differently from the longer fibres discussed above. A ring with radius R can accommodate a fibre of half-length $S \leq R$ without bending. If R decreases and becomes smaller than S , the fibre bends while continuing to touch the ring at either side. Once R has decreased to a value $R_1 = 0.659 \dots S$, the fibre touches the ring at its midpoint. As R decreases further, the fibre is pressed harder against the ring, its curvature radius at the midpoint $R_c(0)$ increases, the kickoff force T increases as the touchdown points slide away. Finally R reaches a value $R_2 = 0.478 \dots S$ where the radius R_c has increased to R . Now the fibre has begun to lie along the ring. A further decrease in R simply expands this contacting region so that the two takeoff points move away from the midpoint of the fibre. The two detached regions then take on the long-fibre shape treated above (figure 1(c)).

It is experimentally easy to confine a straight fibre in a circle of known radius. This geometry could prove useful for making precisely controlled shapes and forces on the macro- and micro-scale. We expect this force-focusing mechanism to be useful where controlled forces need to be exerted on nanometre-scale objects such as surfactant vesicles or biomolecules. The forcing fibre can be, for example, a carbon nanotube [5], a biofilament [6] or a wormlike micelle [7]. The use of different boundary shapes or deformable boundaries [8, 9] adds additional control possibilities.

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